B.A/B.Sc 5th Semester (Honours) Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMH5CC12 (Mechanics-I)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) Prove that for any given system of forces the quantities $\mathbf{X^2+Y^2+Z^2}$ and $\mathbf{LX+MY+NZ}$ are invariable whatever origin, or base point and axes are chosen. [5]

(b) A cycloid is placed with its axis vertical and vertex downwards. Show that a particle cannot be at rest at any point of the curve which is higher than $2a\sin^2\lambda$ above its lowest point, where λ is the angle of friction and *a* is the radius of the generating circle of the cycloid. [5]

(c) (i) Define a catenary of uniform strength. [1]

(ii) Define catenary of uniform strength. Hence prove that it hangs between two vertical asymptotes. [4]

(d) Deduce the expressions for the components of acceleration referred to a set of rotating rectangular axes. [5]

(e) A particle moves on a smooth sphere under no forces except the pressure of the surface. Show that the path is given by the equation $\cot\theta = \cot\beta\cot\phi$, β being a constant ($\theta$ and ϕ are angular coordinates). [5]

(f) Discuss the motion of a particle moving inside a smooth vertical circle. [5]

(g) (i) Explain the concept of momental ellipsoid of a rigid body. [1]

(ii) Show that the momental ellipsoid at the centre of an ellipsoid is given by the equation $(b^2+c^2)x^2+(c^2+a^2)y^2+(a^2+b^2)z^2 = \text{constant}.$ [4]

(h) A uniform heavy circular cylinder rolls down certain distance along a perfectly rough inclined plane in *t* seconds; find the time taken by a hollow circular cylinder of same radius to roll down the same distance along the plane. [5]

2. Answer any three questions: 3×10 = 30

(a) (i) A ladder of weight *W* and length 2*a* rests in equilibrium with its upper end in contact with the smooth vertical wall and the lower end on a smooth horizontal ground. Slipping is prevented by a rope length 2*l* joining the lower end of the ladder to the foot of the vertical wall. Find the tension in the rope by the principle of virtual work. [4]

(ii) Two forces act, one along the line y = 0, z = 0 and the other along the line x =0 , z = c . As the forces vary, show that the surface generated by the axis of the equivalent wrench is $(x^2 + y^2)\, z = cy^2$. [6]

(b) (i) Establish the energy test of stability of equilibrium of a body with one degree of freedom. [5]

(ii) The density of a hemisphere varies as the n-th power of the distance from the centre, show that the centre of gravity divides the radius perpendicular to its plane surface in the ratio $(n+3):(n+5)$. [5]

(c) (i) Show that the differential equation of the central orbit in polar co-ordinates is [5]

$$\frac{h^2}{p^3}\frac{dp}{dr} = F,$$ where the symbols have their usual meanings.

(ii) A particle is projected from the surface of the earth with a velocity $\boldsymbol{v}$. Show that if the diminution of gravity be taken into account, but the resistance of air be neglected, then the path of the particle is an ellipse having the length of the major axis $\frac{2ga^2}{2ga - v^2}$, where a is the earth's radius. [5]

(d) (i) Find the escape velocity of a particle moving under a central force. [5]

(ii) A particle is projected from an apse at a distance c with a velocity $\sqrt{\frac{2\mu}{3}}c^3$. If the force to the centre is $\mu(r^5 - c^4 r)$, then find the path. [5]

(e) (i) Show that the moment of momentum of a rigid body of mass M about a fixed point O, moving in two dimensions is equal to $Mvp + MK^2\frac{d\theta}{dt}$, where the symbols have their usual meanings. [5]

(ii) A rod of length $2a$ is suspended by a string of length l, attached to one end. If the string and rod revolve about the vertical with uniform angular velocity and their inclinations to the vertical be θ and ϕ, respectively, show that [5]

$$\frac{3l}{a} = \frac{(4\tan\theta - 3\tan\phi)\sin\phi}{(\tan\phi - \tan\theta)\sin\theta}.$$